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SAFETY FACTORS AND RELIABILITY OF LARGE STRUCTURES

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In current engineering practice, the strength characteristics of large structures in service — including the safety factor (SF) — are determined by calculation at the design stage [1, 2] and are checked in tests of models. However, if no allowance is made for the scale factor, the application of model tests to the full-scale object may reduce the safety factor [3-5]. Such reductions may have serious consequences, as is shown by the following examples.

The authors of [6] proposed an original design for a spherical pressure vessel (pressures up to 50 MPa) with a capacity of several thousand cubic meters. The vessel has a multilayered shell, each layer consisting of butt-joined plates. Only the joints between the outermost and innermost elements are welded, to ensure that the vessel as a whole is hermetic. The necessary strength is obtained through friction between the layers. It is known that the use of multilayered or coiled pipes on individual sections of pipelines makes it possible to arrest propagating cracks and reduce the seriousness of accidents. Thus, a multilayered design is more desirable than a single-layer design from the viewpoint of preventing brittle fracture. This has been shown by the numerous tests involving the severe shock loading of pressure vessels [7].

If a vessel does suddenly fail, what is the reason [6]? A simple calculation shows that with a capacity of 5000 m^3, the brittle failure of a vessel holding a diatomic gas at a pressure of 50 MPa would be equivalent to the explosion of 150 tons of trotyl (TNT), i.e. would be a catastrophe. The gravity of such an event and the difficulty of experimentally determining the actual safety factor of a full-scale pressure vessel [6] make it incumbent that the reliability of the structure be carefully substantiated.

Let us discuss how vessel strength was verified in [6]. As an example, we performed calculations for a vessel operating at a working pressure P = 20 MPa. The shell of a vessel with an internal radius R_0 = 5 m has n = 40 layers of steel with a yield point σ_y = 350 mm. The thickness of the welded innermost t_1 and outermost t_n layers is 32 mm, while each intermediate layer has a thickness t = 5 mm. According to calculations performed in [6], the given shell, with a total thickness δ = 254 mm, is equivalent in strength to a monolithic shell with a safety factor for σ_y equal to 1.5. It should be noted that brittle fracture of the given vessel would be equivalent to the explosion of 6 tons of TNT.*

Experimental studies of the strength and reliability of the pressure vessels in [6] were conducted on four hemispherical models. The ratio R_0 of the models and the prototype was 1:33 (models 1 and 2) and 1:17 (models 3 and 4), but the models and prototype were not geometrically identical. If we take the parameter δ/R_0 as the criterion of geometric similitude, then model 1 is found to be closest to the prototype. This model, with δ/R_0 = 5.3% (versus 5.1% for the prototype), has a steel shell of eight layers 1 mm thick. The theoretical and actual breaking pressures coincide but remain 44% below the breaking pressure of an equivalent monolithic shell. With allowance for the foregoing, the results of the model tests leave no doubt as to the reliability full-scale versions of the given vessel if it is designed with a safety factor of 2.4 for ultimate strength. In fact, if we reason that the load-carrying capacity (strength) of a structure is determined unambiguously by its stress-strain state alone, then two geometrically similar objects (the model and the prototype) should fail at the same stresses when loaded in the same manner. However, this has yet to be confirmed experimentally.

It is known from studies of the danger of explosion of geometrically similar steel pressure vessels [8-12] that if the smaller vessel fails with the plastic strain ε_{b1} when subjected to shock loading, then (other conditions being equal) the larger vessel will fail with a strain $\varepsilon_{b2} < \varepsilon_{b1}$. The higher the modeling coefficient k, the lower ε_{b2} up to the transition of the material

*The most powerful atom bombs in existence during World War contained 1 ton of TNT.

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to the elastic region. Failure in this case will generally be catastrophic and of a brittle character (even though the material is still a ductile steel). It is just this type of failure that is experienced by spherical steel vessels [10, 11].

The possibility of a substantial reduction occurring in the strength of geometrically similar pressure vessels with an increase in size was examined in [3-5]. This phenomenon is energy-related, since failure (separation of the vessel into parts) is the result of work done by elastic energy stored up in the vessel while under load.* This energy is proportional to the volume of the material $\sim L^3$ (where L is a characteristic dimension of the object), while the fracture work is proportional to the fracture surface $\sim L^2$. Thus, with an increase in L, elastic energy increases more rapidly than the amount of this energy expended on fracture. This results in the breaking stress σ_b being heavily dependent on L

$$\sigma_b \sim L^{-1/2}. \tag{1}$$

If a model fails at $\sigma_b \approx \sigma_y$ in accordance with (1), then when it is enlarged to full scale, i.e. enlarged by the factor k (with the observance of geometric similitude), the vessel will fail at a stress which is $\sqrt{k}$ times lower. This effect of the scale factor on strength is referred to as the energy scale effect (SE) or strong SE [3]. Neglect of this effect — especially the strong SE — by designers of pressure vessels makes the projected safety factor highly unreliable (and negates it completely within the context of the above energy criterion of failure). The reliability of the vessel itself thus becomes questionable.

Relation (1) is only a necessary failure condition, being a consequence of the balance of elastic strain energy and fracture work. A sufficient condition of failure might be the presence of a defect or a region more heavily loaded than other regions (examples are zones containing stress raisers, Griffith cracks, etc.). Of course, careful design and construction of the vessel and the use of a multilayered shell will improve its strength characteristics and reliability and increase load-carrying capacity. However, in principle, if the object is flawed, it will be weaker than its smaller model. With an increase in the dimensions of the model to full scale, the fracture strain enters the elastic region, and fracture changes from ductile to brittle.

Thus if scale effects (especially strong SEs) are not taken into account in the design of large structures, their theoretical safety factor may be unreliable or — the results of calculations notwithstanding — may be nonexistent. In the latter case, the object is destined to fail sooner or later, and experience shows that this failure will occur suddenly at unexpectedly low stresses, having catastrophic consequences.

Do the above findings mean that, in accordance with (1), the strength of a pressure vessel or other object must necessarily decrease as its size is increased? Not at all. There are ways to circumvent this, and we will discuss them below.

The first approach is to replace a large vessel by N geometrically similar small vessels with linear dimensions that are $N^{1/3}$ times smaller than those of the prototype. In this case, with material costs and the total useful volume remaining the same, σ_b will increase by a factor $N^{1/6}$ (for example, with $N = 10^3$, σ_b will roughly triple). However, such an approach is hardly practicable, since it entails a substantial increase in vessel fabrication and operating costs, in addition to requiring a larger work area.

The problem is considerably easier to solve if the vessel has a cylindrical shape. For such a vessel with radius R and height H, the hoop stresses will be twice the axial stresses — regardless of the value of H/R. It thus becomes unnecessary to change H if the original vessel is replaced by N small vessel-tubes. This means in turn that for the same material costs and total useful volume of the vessel-tubes, σ_b will increase by a factor $N^{1/4}$ and gross volume will increase 10% (if the tubes are compactly arranged). To illustrate, with N = 200, the radius of one vessel-tube r would be 1/14 the radius of the prototype (with R = 5 m and r = 0.36, for example) and σ_b would increase by a factor of four [according to (1)]. Thus, other conditions being equal, replacing a cylindrical pressure vessel by a bundle of tubes would significantly increase the load-carrying capacity and reliability of the structure and make it safer (the vessel-tubes would work and fail independently). The fact that the component parts of such a vessel would be identical would allow their configuration to be optimized for the given service conditions and would permit experimental determination of the actual safety factor. Finally, as the vessel in [6], a vessel of the type just described could be built on site.

A second approach to resolution of the problem being discussed is the use of oriented fiber composites. It was shown in [13, 14] that no role is played by the energy scale effect in the failure of such materials (this applies in particular to glass-fiber-reinforced plastics). The fact is that the main load-bearing elements of a glass-plastic are fibers, the diameter of which d = const is independent of the size of the object. Thus, in accordance with (1) (where $L \equiv d$), σ_b = const. This accounts for the

*Since the time to failure is relatively short, the work done by external forces will be unimportant in the given case.

absence of an energy scale effect. Whereas for metallic structural materials the most dangerous event is brittle fracture in the elastic strain region — where fracture toughness K_{Ic} is low — elastic deformation and brittle fracture are natural phenomena for an individual glass fiber. In this case, even the failure of a large number of fibers simultaneously is not catastrophic for objects made of fiber composites [14]. In accordance with (1), for glass fibers in particular it is possible to substantially increase σ_b by decreasing d [3, p. 75], despite the negligibly low K_{Ic} of glass (compared to steel). For example, for glass fibers VM-1 with $d = 10\ \mu m$, $\sigma_b = 4.2$ GPa [15], i.e, σ_b is considerably higher than for steels. Here, the density of the glass is one-third the density of steel.

Some observations must be made in regard to structures made of multilayered and coiled materials. Their main load-bearing element is a layer whose thickness t = const, regardless of the overall size of the object. In terms of their reaction to loading, these materials are similar to composites (particularly glass-plastics) having fibers with d = const. They differ from composites in two important respects.

1. For a structure to function properly, the tensile stresses in it should not exceed σ_y. The value of σ_y is low compared to the σ_b of glass fibers and, in contrast to such fibers, has an upper bound. For glass fibers, the value of σ_b can be increased significantly by decreasing d.

2. Due to the absence of a matrix, the layers of material are acoustically coupled. In a glass-plastic, conversely, there is almost no coupling of the fibers and matrix (their acoustic resistances differs by approximately one order). Thus, as in monoliths, the energy scale effect may be manifest in multilayered and coiled materials. However, this effect will be considerably weaker than in a monolith [16].

According to data from numerous studies [3, 8, 12, 16, 17], two failure conditions — necessary and sufficient — are satisfied in the case of severe shock loading. According to [17], a fourfold increase in the diameter of geometrically similar one-layer steel tubes would lead to a decrease in ε_b by a factor of 2-2.5 if the tubes were subjected to blast loading. Similarly, it was reported in [16] that a tenfold increase in the diameter of a coiled tube would decrease ε_b by a factor of 1.5. Moreover, since a decrease in relative elongation with an increase in specimen size has been observed in static tests [3, p. 74], similar results might be expected to be seen under static loads in service. Thus, returning to [6], we should keep in mind that even if layer thickness remains unchanged and the number of layers is increased by the factor 33×8, when the results of tests with model 1 are applied to a full-size vessel, the scale factor might cause ε_b to decrease to the elastic strain region, i.e., might result in brittle fracture.

Consequently, it is improper to evaluate the strength of large objects on the basis of results of model tests without allowance for the scale factor: even with the use of a multilayered structure and a large safety factor, manifestation of the energy SE can significantly reduce or completely eliminate the actual safety factor of the object.

Current methods of guaranteeing the necessary safety factors in large structures are based on the selection of certain materials for the load-bearing elements. Embrittlement of the material accompanying an increase in the size of the object is accounted for on the basis of results of fracture-toughness tests of full-thickness specimens or displacements of the critical brittleness temperature with a change in specimen dimensions. However, as was shown in [18], neither do tests such as these guarantee safety against brittle failure. The use of a multilayered structure also does not necessarily solve the problem: on the one hand, such materials cannot be used in certain applications; on the other hand, as was shown above, they are not free of the scale effect. It is recommended that oriented fiber composites be used in such applications, but there is one more approach that can be taken when this is not possible or when traditional metals must be used.

Assuming that the necessary and sufficient fracture conditions are met in the case of severe shock loading, we can evaluate the fracture conditions for the full-scale object by loading 2-3 models of different sizes to failure. Let us illustrate this by using the example of the explosive failure of geometrically similar spherical steel vessels. For simplicity, we will write the equation describing its deformation in idealized bilinear form

$$\sigma = E\varepsilon \quad \text{at} \quad \sigma \leqslant \sigma_0,$$
$$\sigma = \sigma_0 + K(\varepsilon - \sigma_0/E) \quad \text{at} \quad \sigma > \sigma_0,$$

where σ_0 is the idealized yield point; E and K are the elastic and strain-hardening moduli. If the radius of the vessel R is small enough so that part of the kinetic energy of the vessel is expended on plastic flow before fracture, then it is not hard to find the relationship between R and ε_b:

$$R_y/R = [(1 - K/E) + K\varepsilon_b/(E\varepsilon_y)]^2. \tag{2}$$

Here, ε_y and R_y are the strains at the yield point and the vessel radius in the case of fracture at the yield point. When the vessel fractures in the elastic region (K = E), Eq. (2) takes the form

$$R_y/R = (\varepsilon_b/\varepsilon_y)^2. \quad (3)$$

Equations (2) and (3) make it possible to find the fracture strain ε_{b1} of a test model of radius $R = R_1$ and, with known E, K, and ε_y, evaluate the limiting radius R_y; at $R > R_y$, the fracture of the vessel will be brittle in character. The effect of strain rate and material temperature on σ can probably be ignored in this case as a first approximation, considering the fact that these factors yield small corrections of different signs. For example, as was shown in [19], for vessels of steels 25 and 22k with $\delta/R \approx 0.2$, the estimated value $R_y \approx 0.2$ m. This result is consistent with the experimental data.

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